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Mechanical vibration — Evaluation of measurement results from dynamic tests and investigations on bridges

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Foreword

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ISO 18649 was prepared by Technical Committee ISO/TC 108, *Mechanical vibration and shock*, Subcommittee SC 2, *Measurement and evaluation of mechanical vibration and shock as applied to machines, vehicles and structures*.

Mechanical vibration — Evaluation of measurement results from dynamic tests and investigations on bridges

1 Scope

This International Standard provides methodology for the evaluation of results from dynamic tests and investigations on bridges and viaducts. It complements the procedure for conducting the tests as given in ISO 14963 and considers

- the objectives of the dynamic tests,
- the techniques for data analysis and system identification,
- the modelling the bridge,
- the evaluation of the measured data.

The evaluation may seek to define all of the dynamic characteristics of each mode of vibration examined, i.e. frequency, stiffness, mode shape and damping, and their non-linear variation with amplitude of motion. These can supply information on the dynamic characteristics of a structure for comparison with those assumed in design, or as a basis for condition monitoring or system identification. The dynamic tests considered in this International Standard do not replace static tests.

This International Standard gives guidance on the assessment of measurements carried out over the life cycle of the bridge. The stages of the life cycle that are considered are

a) during construction and prior to commissioning;

b) during commissioning trials;

c) during specified periods throughout the life of the bridge;

d) immediately prior to decommissioning the bridge.

Figures 1 and 2 illustrate the relationships between the various stages involved in vibration monitoring.

This International Standard applies to road, rail and pedestrian bridges and viaducts (both during construction and operation) and also to other works provided that they justify its application. The application of this International Standard to special structures (cable-stayed or suspension bridges) requires specific tests that take into account the particular characteristics of the work.

NOTE “Bridges and viaducts” are called “bridges” throughout this International Standard, and “viaducts” is used if it is necessary to distinguish.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2041, *Vibration and shock — Vocabulary*

ISO 14963, *Mechanical vibration and shock — Guidelines for dynamic tests and investigations on bridges and viaducts*

ISO 14964, *Mechanical vibration and shock — Vibration of stationary structures — Specific requirements for quality management in measurement and evaluation of vibration*

3 Terms and definitions

For the purposes of this International Standard, the terms and definitions given in ISO 2041 and the following apply.

3.1
buildability
property of a structure that enables construction to proceed in a safe, timely and economic fashion

NOTE The buildability of bridges may require construction to proceed in a strong wind, so wind effects on vibration may need to be monitored.

3.2
environmental compatibility
environmental impact on a new bridge, involving wind effects, air noise and ground vibration, which may need to be evaluated

3.3
serviceability
limit state beyond which a structure is not satisfying the operating requirements such that it is no longer fit for purpose

3.4
monitoring
programme of measurements, usually over a period of time, whereby changes in an appropriate parameter may be interpreted as indicating a change in the state of the structure

NOTE It is important to establish a benchmark and allow for changes attributable to cyclic environmental factors such as diurnal or seasonal changes of temperature and humidity.

3.5
running safety
property whereby traffic crossing a bridge at an appropriate speed is not deleteriously affected in maintaining direction or stability

3.6
riding quality
property whereby occupants of vehicles crossing a bridge at appropriate speed are not exposed to such levels of vibration as to adversely affect their comfort

4 Vibration measurement

4.1 General considerations of vibration measurement

The guidelines for vibration measurements as given in ISO 14963 shall be observed and the quality requirements for these measurements as given in ISO 14964 shall be fulfilled. Measurements may be carried out on bridges under construction and in commissioning and on bridges in service.

4.2 Monitoring of a bridge during construction and for commissioning

4.2.1 Objectives of vibration monitoring

The objectives of vibration monitoring shall be specified as follows (see also Figures 1 and 2):

- evaluation of the accuracy and buildability of construction,
- evaluation of structural performance during construction and upon completion,
- assessment of safety of bridge during construction and upon completion,
- evaluation of serviceability upon completion,
- evaluation of environmental compatibility,
- determination of the initial characteristics of vibration for maintenance and for the calibration of the numerical model of the bridge in service,
- feedback to structural design.

Uncertainly of results in each process of measurement and evaluation can not be avoided and there is a possibility to include uncertainty as shown in Figure 1. So reduction and qualification of measurement uncertainty and error are needed in the process.

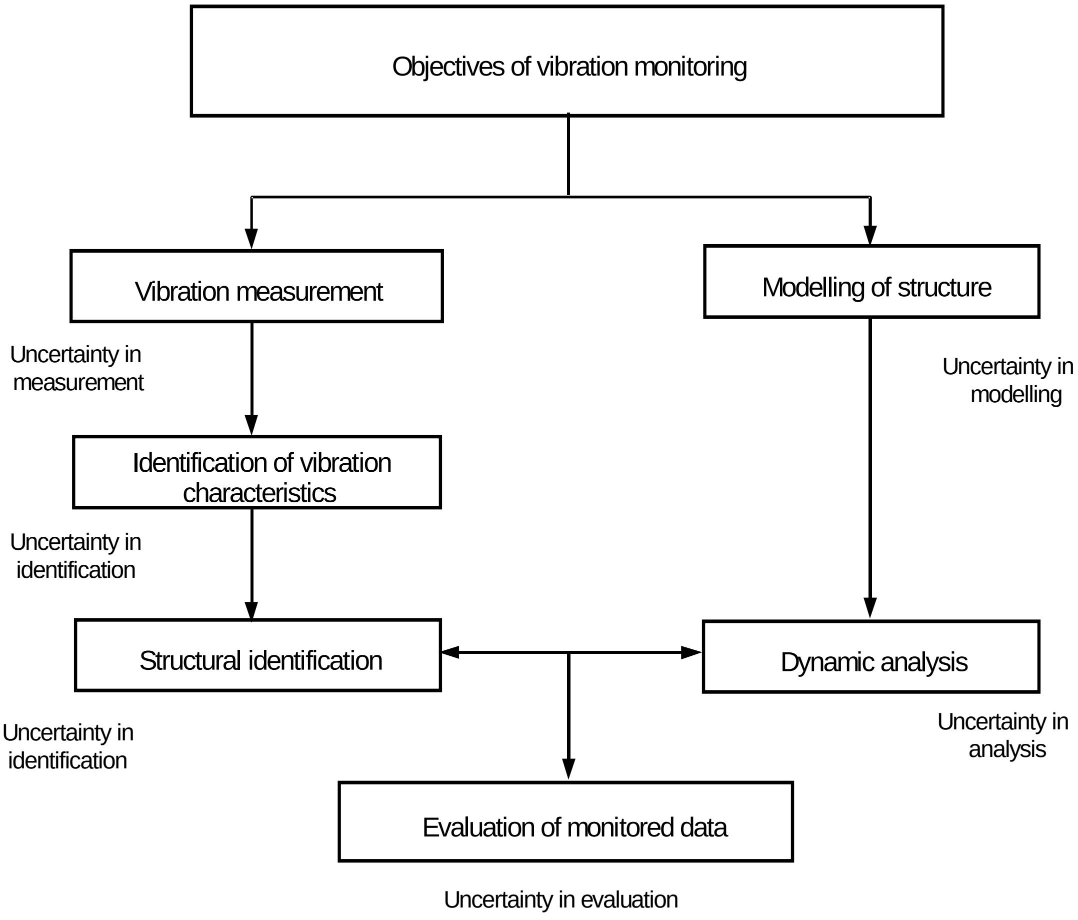

Figure 1 — Flowchart of bridge vibration monitoring

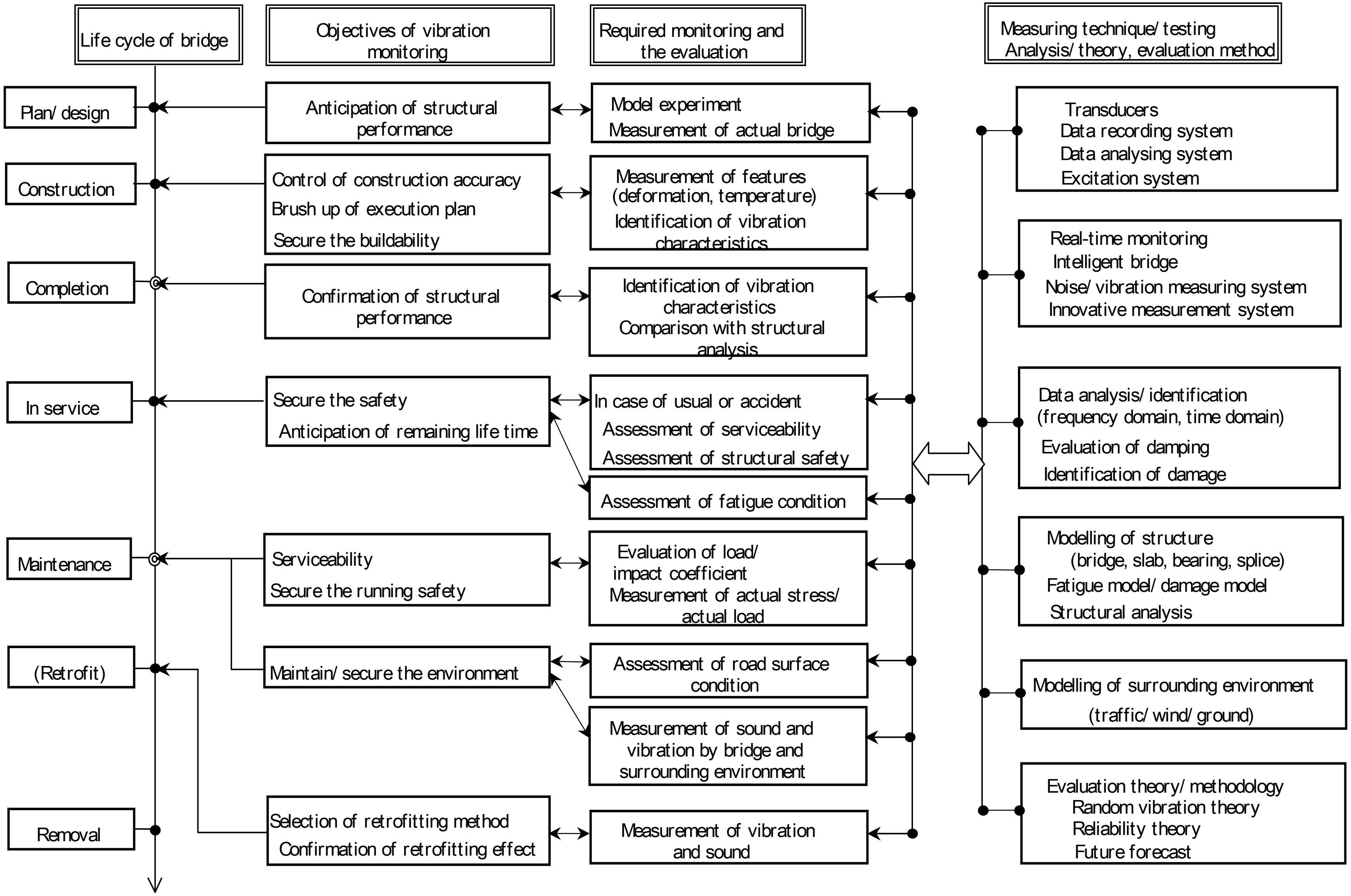

Figure 2 — Overview of bridge vibration monitoring

4.2.2 Evaluation of construction management

4.2.2.1 General

Vibration measurements on bridges may be conducted during construction. For example, vibration tests on cables of cable-stayed bridges or suspension bridges are used to control the tension of the cables. In order to control the profile of the bridge in construction, measurement of the vibration of cables is required. Dynamic measurements may also provide an indication of when high vibration levels will have an adverse effect on construction.

4.2.2.2 Evaluation of cable tension

Dynamic characteristics are greatly influenced by the support conditions. Cable tension of a cable-stayed or suspension bridge is one of the main parameters for construction management. Vibration of cables is easily measured for the determination of the natural frequency of transverse vibration. This depends upon cable tension and is given by a well-known equation. In this case, the numerical model will need to consider bending rigidity and the end support of the cables.

4.2.2.3 Evaluation of buildability of construction

Vibration measurements can provide the required information to determine when construction work is either unsafe or the quality control is likely to be adversely affected. If the bridge vibration, and wind and earthquake excitation are continuously monitored the decision can be made when an allowable limit is exceeded.

4.2.3 Characteristics for the evaluation of structural performance

4.2.3.1 General

Natural frequency, damping and dynamic response of the structure and the surrounding area and sound propagation from/through structure are measurable characteristics which can be used for the evaluation of structural performance.

4.2.3.2 Natural frequencies and mode shapes

The natural frequency and its mode shape are easy parameters to measure. The supporting conditions and temperature of the structure are major factors influencing natural vibration; hence they should be monitored before and after construction. Geometrical non-linearity of flexible bridges and material non-linearity of superstructures on substructure are aspects to be considered. These aspects are listed as follows:

- natural frequencies,
- modal shapes,
- movements of shoe and boundary condition of structures,
- geometrical non-linearity effect of structure,
- material non-linearity of ground,
- effects of isolator and vibration control devices,
- effects of temperature.

NOTE Isolator and control devices to reduce vibration may also introduce non-linearities.

4.2.3.3 Damping

The damping coefficient, or logarithmic damping ratio, can also be measured. The measurement of damped free vibration produced by stopping the forced vibration provides a direct measurement of damping characteristics, at least for the fundamental mode. Amplitude and temperature dependencies are important factors for damping measurement. It may be necessary to consider the effects of support condition and isolation devices. When damping characteristics are required for large-amplitude motion, forced vibration tests that generate high-amplitude vibration are appropriate. Evaluation for strong earthquakes or wind may require damping values for large-amplitude motion.

Elements affecting the damping characteristics of bridges are as follows:

- aerodynamic and hydrodynamic effects,
- connections and joints,
- bearings and shoes,
- pavement (rheology of materials),
- effects of substructures,
- effects of the foundation.

The amplitude dependencies of frequencies and damping characteristics of bridges require careful analysis of the data. Different damping characteristics will be provided by different structural types and in different locations, so the overall damping effect is the integral of these elements.

4.2.3.4 Characteristics of the dynamic responses of a structure with the surrounding media

The measurement of dynamic response may involve strain, acceleration, velocity or displacement. It is also important to consider boundary conditions. The results from ambient vibration tests or impact tests may not be appropriate for some dynamic response evaluations because of the small amplitude of the loading. Using forced-excitation tests, resonance response curves can provide data for larger amplitude motion. Accurate analysis of ambient vibration for small amplitude may be suitable for structural health monitoring of bridges. Tests using moving vehicles can give the dynamic response related to the speed and pattern of vehicles. Fatigue analysis requires dynamic response as stress range histogram. The points to be considered are as follows:

- accuracy of ambient vibration analysis,
- impact test for dynamic property of surrounding media,
- effects of water or tidal flow,
- excitation method.

4.2.3.5 Sound radiation around/through structure

Microphones placed on the surrounding ground can detect the sound radiation from bridges due to moving vehicles. Characteristics of sound propagation are used to evaluate environmental effects on the surrounding area. Parameters to be measured are as follows:

- sound level,
- sound frequency,
- traffic density,

- traffic speed,
- types of vehicles,
- impulsive effects,
- roughness of road/track surface,
- ground stiffness and its interaction with substructure.

4.2.4 Assessment of safety during construction and upon completion

4.2.4.1 Confirmation of design for earthquake

For safe construction in a highly seismic area, vibration monitoring is needed. Depending upon the data, engineers can assess the risk during construction, and this may influence the construction. Data on vibration under severe loading conditions are important. The assessment is based on

- characteristics of natural vibration and its damping,
- dynamic response characteristics,
- reinforcement of structure,
- isolation system on bridge,
- diagnosis of structural health after disaster.

In the design process for earthquake performance, the numerical model for dynamic response is constructed by the combination of total/part of superstructure used in static design and the substructure including basement and the surrounding ground. These data should be utilized in the evaluation analysis.

Measurement of natural vibration of substructure after its construction and the non-linear vibration properties of the ground should be taken into account. Evaluation of damping characteristics is accomplished by the comparison of measured data with the assumed values used in design process. Support condition and amplitude dependency should also be taken into account. The effect of temporary structures and the pavement on vibration properties should also be considered.

4.2.4.2 Confirmation of design for wind

The dynamic response for wind can be measured and compared with assumed values. Assumed values may be obtained through experiments in a wind tunnel as a part of structural design process. Measured data may include the effects of velocity and direction of wind and amplitude dependency. After analysing all these effects, damping devices may be considered.

4.2.4.3 Confirmation of fatigue design

Fatigue design considers the dynamic stress range of members and the number of cycles encountered. In this case, the stress range is given by the sum of static stress and the coupling effect with moving vehicle. Monitored data for actual stress should be compared with the assumed values used in fatigue design. The dynamic amplification factor to amplify static stress range is used and it depends on the road/track profile and travelling pattern of traffic load. The coupling effects with vehicles are needed to monitor the structural health of the bridge. Non-stationary vibration due to irregular undulation of surface of track and road can be important.

4.2.5 Serviceability on a completed bridge

Vibration perception of pedestrians, vibration effects on moving vehicles and the comfort of passengers are part of potential serviceability problems. Vibration monitoring is undertaken to evaluate these effects and the design should be checked and necessary measures should be considered.

In the evaluation of vibration perception of pedestrian the amplitude of dynamic response as well as frequency of vibration are considered. In the evaluation of the effects on moving vehicle and comfort of the passengers the amplitude of dynamic response on the floors and wheel axles of vehicle is take into account.

In the comparison between measured data and numerical results from modelling the moving loads, coupling vibration effects should be taken into account. Bridge vibration due to moving vehicles and the comfort of passengers is also the problem of serviceability.

4.2.6 Evaluation of environmental compatibility on a completed bridge

Environmental vibration, noise and change of wind direction are taken into account in the evaluation of environmental compatibility. Monitored data is used to analyse these effects and is compared with dynamic characteristics of structure. Necessary modifications may be required depending on the results. Numerical simulation of the propagation of ground vibration and sound radiation is used to identify the level of those effects.

4.2.7 Determination of the initial vibration characteristics on a completed bridge

Long-term monitoring will start after construction and initial values of vibration characteristics are required to monitor changes in parameters due to deterioration or damage. The effects of deterioration or damage on the vibration characteristics are generally small, so an effective method to extract the required information of damage should be used. Local excitation and the application of beating phenomenon due to those small differences of modal parameters are useful methods.

4.2.8 Feedback to overall performance

Data given through the above mentioned evaluation should be fed back to design engineers to apply in future design. Classification of the data is also helpful when the data is used in future for design of all types of bridges.

4.3 Monitoring of a bridge in service

The objectives of vibration monitoring of the bridge in service are:

- evaluation of travelling load,
- evaluation of structural performance,
- evaluation of wind and hydrodynamics,
- assessment of safety,
- assessment of serviceability,
- assessment of environmental compatibility.

Normal and emergency monitoring on bridge vibration are used depending on the maintenance management of bridge. Detailed analysis to identify the damage and defect is needed. Traffic condition and roughness of road, rail surfaces, wind and hydrodynamic effects will have a significant impact on the fatigue stress. Dynamic effects should be monitored through measurement.

5 Data analysis and method of structural identification

5.1 General

Structural identification aims to provide a good correlation between the numerical model of the bridge and experimental measurements. From the model, which may be based on finite elements, the modal parameters are identified, primarily the modal frequencies and associated mode shapes. The same parameters can be identified by experiment, which should also determine the damping value for each mode. The identification methods may adopt either time domain or frequency domain procedures depending on the problem.

5.2 Data analysis and domain

Data analysis may be conducted in either the time domain or the frequency domain or both domains. Depending on the problem, the engineer should decide which domain to use. In clause A.1 the relationship between the time and frequency domains is shown.

In the analysis of vibration data, statistical distributions of stress, velocity, acceleration and displacement are obtained. The distributions of stress and displacement are used for condition evaluation of bridges.

5.3 Consideration on digitizing

Data obtained from experiment are usually digitized from the analogue signal using an analogue to digital (A/D) converter. The selection of the sampling frequency or time step for the digitization procedure is very important, and care should be taken in order to retain the required accuracy from the analogue data. The following are main topics to be observed.

a) Error in A/D transformation

The sampling frequency of the A/D converter should be carefully specified and a resolution consistent with the objective of the targeted natural frequency is recommended.

b) Misreading of peak values

Both in the time domain and the frequency domain, there is a possibility that the true peak values may not be identified during digitization. Hence the shape of the transfer function as well as peak values should be considered in the identification of frequency and damping when using the half-power bandwidth method.

c) Resolution of FFT

In the Fourier transformation of the digitized data it is impossible to get higher frequencies than those specified by the time interval of digitized data Δt. This limiting frequency is called the Nyquist frequency [15] and expressed as $f_N = 1/(2\Delta t)$. The frequency resolution Δf is given by the inverse of the total time T as $\Delta f = 1/T$.

5.4 Identification of vibration characteristics in the time domain

5.4.1 General

Natural frequency, modal shape and damping coefficient should be identified when determining the vibration characteristics of a system. It is recommended that damping coefficient is identified in the time domain. If the non-linearity and amplitude dependency are significant, the analysis should be performed in the time domain.

In the time domain, ideally one mode should be considered at a time, which may require the data to be extracted from the measured data by filtering. There are situations where closely spaced frequencies exist depending upon the type of structure and constitution of members. Here, it will be difficult to extract data for one mode and the identification method for closely spaced modes system must be used.

5.4.2 Extraction of single natural frequency component

Measured data include generally many vibration modes and it is difficult to identify vibration characteristics accurately in the time domain. Ideally single-mode data should be extracted from the measured data (see clause B.1). The frequency may be identified by the following:

- transformation of the measured data to the frequency domain,
- filtering to extract single-mode data,
- inverse transformation from frequency domain to time domain.

Low-pass, high-pass and band-pass methods can be used for filtering the data, depending upon the circumstances.

5.4.3 Natural frequency

Identification of the natural frequency of a single mode from the time history of the signal may consider the time period between the following:

- the peak responses,
- zero-crossings.

If the bridge has isolation devices or the support has amplitude dependency, the natural frequency will usually vary with amplitude of motion. In this case extracted single-mode data should be inverse transformed to time domain to compare those vibration amplitudes with measured data.

5.4.4 Natural frequency mode

If the damping is small and the system is considered as a proportionally damped system, the relative displacement vector of natural frequency mode is constant and not dependent on time. In this case the frequency mode is obtained by plotting relative amplitude as normalized values. If the bridge has large damping devices, the vibration characteristics will exhibit non-proportional damping. In this case the measured mode has a phase shift and the modal shape changes even in one period of vibration. It is difficult to identify this in the time domain.

5.4.5 Damping

Damping coefficient ζ is identified by natural vibration data of single mode in the time domain (see Figure 3). The logarithmic decrement δ is obtained by

$$\delta = \ln \frac{x_{i-1}}{x_i} \tag{1}$$

and the damping coefficient is given by

$$\zeta = \frac{\delta}{2\pi} \tag{2}$$

Measured damping coefficients may vary depending upon the effect of transient vibration. Hence averaging over the different parts of waves, or piecewise waves of different amplitude should be used in the identification. The curve fitting method adopting a non-linear least square approach is used for identification of the damping coefficient [15].

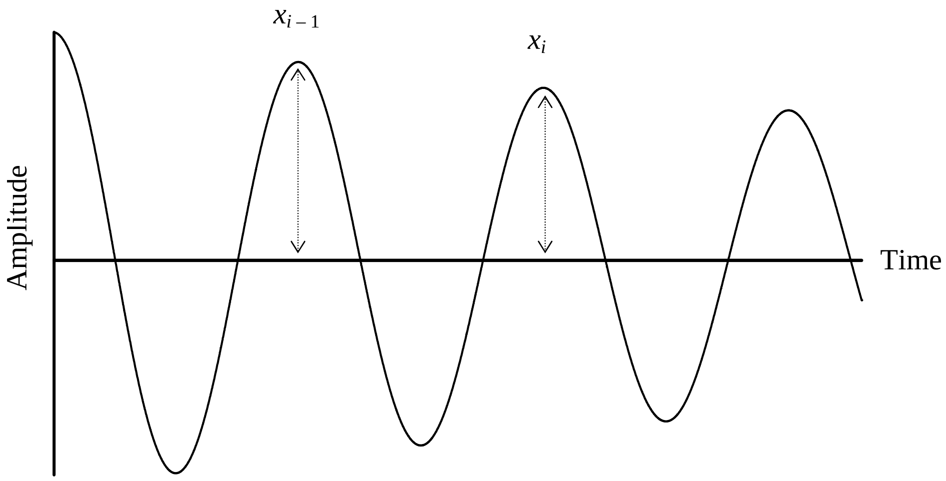

Figure 3 — Identification of the damping coefficient in the time domain

5.4.6 Identification of vibration characteristics with closely spaced modes

In large scale structures with a variety of structural types, there may be modes with closely spaced frequencies.

EXAMPLE

a) Suspended slab bridge: The first symmetrical mode is close to first anti-symmetrical mode depending upon the sag-span ratio.

b) Cable-stayed bridge: In multicable type cable-stayed bridges, closely spaced frequencies often occur between the total vibration system and cable system.

c) Bridge which has a tuned mass damper: Here the vibration beating phenomenon is seen and it is difficult to identify the vibration characteristics by the above method.

The methods recommended for these cases are as follows:

- estimation by using beating wave data as the resultant of two single modes,
- extraction of the separate frequencies by superposing the beating data with weighted function,
- curve fitting method,
- EK-WGI method (extended Kalman filter).

5.4.7 Random decrement technique (RD method)

With natural or ambient vibration the expected value of random excitation force should be considered to be white noise. In order that the superposition of certain numbers of wave data yield a meaningful component of a natural frequency, care should be taken with regard to amplitude dependency and the band-pass filter method used.

5.5 Identification of vibration characteristics in the frequency domain

There are some advantages when frequency domain analysis is used:

- The frequencies are clearly seen in the transfer function and power spectrum,
- modal characteristics can be identified using modal analysis,

- multiple transfer functions are obtained as the impulse response function to relate input to output effect,
- statistical analysis is easily applied by assuming a stationary process,
- non-stationary (impulsive) vibration effects need to be carefully considered to extract these parameters.

However, disadvantages are:

- Analysis is impossible in the case of non-linear system and time-dependent system,
- depending on the problems, accuracy may be decreased.

But non-stationary spectrum analysis such as wavelet analysis may avoid some of these disadvantages. Calculation of the transfer function from measured data is shown in clause A.2.

5.6 Structural identification and inverse analysis

In the theory of system identification, the structural identification method is applied and a mathematical model is identified to relate the excitation (input) to vibration response (output) (see Figure 4). The accuracy of the mathematical model is evaluated using the object function of error between mathematical model and real structure. The characteristic matrices are defined as the coefficient matrices of the governing equations of motion with multi-degree of freedom as mass, damping and stiffness matrices. Modal parameters are identified by using complex eigenvalue analysis on those governing equations (see clauses A.3 and B.2).

For higher-frequency modes, many measurement points are needed to measure the mode shape. The application of structural identification and inverse analysis can be used to identify the vibration characteristics of the data from a limited number of measurement points. Stochastic error function analysis to evaluate the accuracy of identified parameters is also useful.

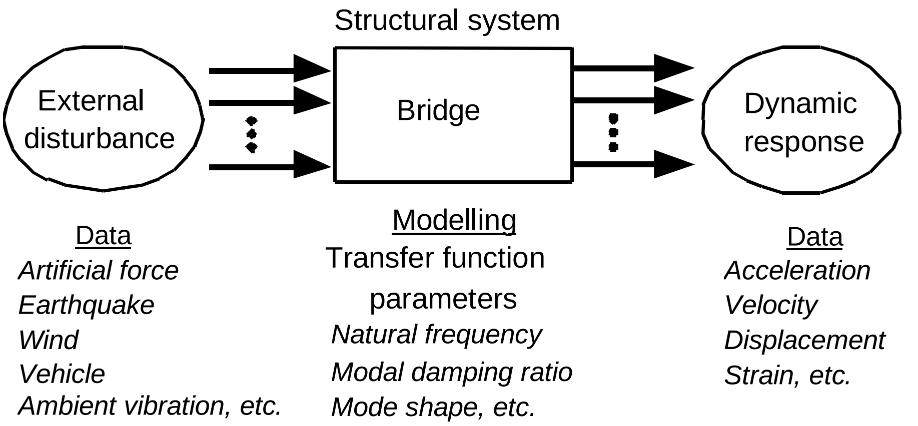

Figure 4 — Relationship between input and output of a structural system

6 Modelling bridges and their surrounding environment

6.1 Modelling bridge structures

A bridge structure is a three-dimensional structure which has beam, plate, cable, pillars, foundation and other elements. To model those geometrical structures, structural and material characteristics of members should be taken into account. Generally it is recommended that the appropriately detailed model should be used in the analysis. Depending upon the problem to be modelled simplified and effective beam or plate theory may be applicable. In dynamic analysis by finite-element method (FEM), there are several methods to model the mass of a structure like lumped or consistent mass models.

Bridge types are classified as follows:

- girder bridge,
- truss bridge,
- framed bridge,
- arch bridge,
- suspended slab bridge,
- suspension bridge,
- cable-stayed bridge.

Those bridges are generally supported on several piers and abutments to transfer forces to the ground. If the width of bridge is small compared to its longitudinal length, the bridge structure is modelled using plane or space frame elements in dynamic analysis. Axial, bending and torsional rigidity of those frame elements are taken into account in the numerical analysis. Recommendations for modelling the different types of bridge structures are as follows.

a) Girder bridge

Girder bridges consist of main girders with open cross section or closed cross section (box), cross beam (floor beam) to connect the main girder and slab. If the width of bridge is small compared to its span, the bridge can be modelled by beam elements considering bending vibration. In case of curved girder bridges, the coupling effect between bending and torsional vibration should be included.

b) Truss bridge

Truss bridges consist of spatial frame elements and the connections of elements are considered as pin (hinge) joints to take only axial deformation of members into account. Spatial truss bridges usually have many members, so that reduction or simplification of the number of degrees of freedom in the numerical modelling should be considered. The effective beam theory, replacing spatial structure with simple beam elements is a useful technique. Another simplification is to use the model of spatial mass system. The stiffness of one panel of a spatial truss is modelled using effective box elements and the mass of the panel is taken into account by concentrating the mass at the centre of the box element.

c) Framed bridge

Axial rigidity of the members of framed bridges is not negligible. Out of plane and torsional deformation should be taken into account in three-dimensional analysis of dynamic response. If a framed bridge is supported on weak ground, the effect of its support conditions should be considered in the numerical modelling. Damping may be contributed by the support condition, so the modification of ground or surrounding environment is important.

d) Arch bridge

In the modelling of arch bridges, polygonal approximation by using straight elements is useful. The effect of connection between arch (including lower chord) and vertical stiffening member is considered as pin (hinge) or rigid joint depending upon the problem. Geometrical non-linearity of arch members is not negligible in some cases. In this case the stresses and displacements are different from the results obtained using small deformation theory. If the span of arch is small, small-deformation theory is applicable.

e) Suspended slab bridge

Suspended slab bridges have thin slabs and cables, and may encounter large deformations. Hence wind-induced vibration and serviceability for pedestrian may need to be considered. If the sag-to-span ratio f/l is large, the numerical model should include these appropriate coupling effects. Three-dimensional modelling is recommended to include the torsional modes of vibration.

f) Suspension bridge

Vibration behaviour of suspension bridges is considered in two systems depending upon the problem to be solved. One of the vibration systems consists of the combination of cables, stiffening girders and vertical hangers and the another is the complete vibration system including tower, pier, foundation and its surrounding environment. The first is mainly used for performance response analysis in wind load and dynamic response analysis under traffic load, the second is used mainly for performance under earthquake. Cable and structural elements of stiffening girder have different vibration characteristics. Then its dynamic behaviour is complicated and the system has coupling effects between vertical and torsional modes. Finite-deformation theory is normally used to model these suspension bridges.

g) Cable-stayed bridge

Cable-stayed bridges consist of towers, inclined cables, stiffening girders and substructures. The elements have different vibration characteristics and coupling vibration occurs. Sometimes coupled vibration between cables and girder is significant. High-frequency modes have strong coupling effects between them. Three-dimensional modelling is recommended. For the performance assessment of wind resistance, simplified modelling may be used.

6.2 Modelling traffic loads

6.2.1 Modelling of vehicles

In the analysis of coupling vibration between bridge and vehicle, there are several methods of vehicle modelling. Depending upon the problem to be solved one-degree-of-freedom (1-DOF), two-DOF (2-DOF) and multi-degree-of-freedom (M-DOF) models are used for vehicle modelling. To fit measured results with the numerical simulation, vehicle modelling is important. If the coupling effect is taken into account, the effect of continuous series of vehicles should not be neglected. If the effect of continuous traffic vehicles is considered, dynamic amplification effect will be reduced from the results for a one-vehicle model in numerical simulation. The distribution of distances between vehicles is also a factor to be considered.

For the modelling of a railway vehicle, a multi-body vibration model composed of carbody, truck and wheelset connected with springs and dampers should be used. The bogie-type train and articulated-type train should be distinguished in modelling. Number of vehicles in a train, length of train and wheelbase are factors to be considered in the calculation of the dynamic amplitude of the bridge response.

6.2.2 Surface roughness

The effect of surface roughness of the track is not negligible when considering the coupling effect of the vibration between bridge, track surface and vehicle. ISO 8608 can be used to evaluate the coupling effect.

6.3 Modelling of human walking and its dynamic effect

Walking models and the dynamic load on the bridge are used to evaluate the vibration serviceability of pedestrian bridges. Several methods for modelling the walking load are shown in Annex C.

Horizontal vibration mode of pedestrian bridge of suspension type is accelerated by the coupling effect between (half the) step frequency and natural frequency of horizontal transverse mode of bridge. In this case lateral load model is useful to evaluate those coupling effects [13, 14, 22].

6.4 Wind load

Wind load for bridge is taken into account in static and dynamic behaviour as

a) Static

  - static deformation by static air pressure,
  - unstable phenomenon by static air pressure (divergence, horizontal buckling);

b) Dynamic

  - vortex flow vibration,
  - galloping vibration and flutter vibration.

The vibration due to fluctuating air pressure or non-stationary varying air pressure depends upon the characteristics of natural wind. If the effects of wind are to be considered in evaluation, the variation characteristics of wind velocity should be taken into account in following aspects:

- standard deviation of wind action,
- power spectrum density of wind velocity,
- spatial scale of random variation,
- spatial correlation of wind velocity,
- vertical angle of wind,
- frequency distribution of velocity.

6.5 Modelling of ground for viaduct vibration

In the analysis of coupling vibration between viaduct and surrounding ground, modelling of ground should consider the ground characteristics which may include stratification, embedded structures and other systems. But the modelling of assuming semi-infinite uniform elastic half-space of three-dimensional coordinate is normally used for simplification.

7 Evaluation of monitored data and its application

7.1 Method of evaluation and evaluation criteria

7.1.1 Classification of evaluation

Measured data are analysed in order to yield information on bridge performance, and quantitative or qualitative decisions made based on this information. Evaluation of monitoring data is undertaken for safety, integrity diagnosis, serviceability and environmental effect of structures. In the evaluation process, reliability and propriety of structure should be clarified, so that standards can be established as a basis for evaluation. Generally as an evaluation standard, reasonable or standardized values or limit state values of evaluation parameters are used.

As the standard values for evaluation will be different for different situations, they should be divided into categories. In some cases evaluation will be undertaken for a combination of categories. If the evaluation is qualitative and not quantitative, comparable evaluation will be undertaken in the manner described for the following four categories.

Category 1: When values of limit states exist. The basis of the evaluation is to check whether the monitored values exceed the limit state.

Category 2: When defined values or defined ranges of condition exist. The basis of the evaluation is to compare the monitored data with those defined values or ranges.

Category 3: When estimated values for healthy condition or similar condition of structure exist. If the evaluation values for limit states or defined states are not clear, the values which are estimated numerically for the case of healthy condition or similar condition are useful. In this case the basis of the evaluation is to check whether the monitored data exceed those estimated values or they are within the range of estimated values with acceptable error.

Category 4: When monitored data for previous condition or numerical values for similar conditions exist. If previously monitored data or data for an assumed condition are available, the measured data can be compared with those data to evaluate how much the structure improved or deteriorated.

7.1.2 Factors to be considered in evaluation

The following factors should be considered in evaluation:

a) Measurement uncertainty in monitoring data;

b) variability of structural characteristics to be evaluated;

c) discrepancy between measured (monitored) state and evaluated state when standard values are calculated as evaluation parameters. The differences between actual and standard conditions should be taken into account. As an example it is recommended that the error between measured and estimated natural frequency of typical mode should not be over a given percentage.

7.2 Evaluation during construction

For the evaluation of bridge vibration during construction, limit values of vibration are given by authorities. Amplitude, velocity, damping and frequency of vibration are major indicators. The manager of construction should utilize the monitoring data to decide how to manage the construction.

Damping devices may be needed if the vibration exceeds the limit. Assessment of these devices should be undertaken by evaluating the measured data.

7.3 Evaluation of structural safety in service

7.3.1 Evaluation of damage by vibration monitoring

Generally if damage increases on a structure, the lower natural frequencies decrease and damping coefficients increase. As these changes are small, it is difficult to identify the damage accurately. Measurements on the part of structure where the probability of damage is high, may help to detect local damage through the analysis of the dynamic response.

As the damage propagation within bridges is generally slow, depending on the external conditions, monitoring of vibration characteristics over the long term is helpful for the evaluation of structural health. The general tendency for the change of structural characteristics as damage propagates can be evaluated.

7.3.2 Confirmation of modification effect by vibration monitoring

When a bridge is modified or repaired and its vibration monitored, the evaluation of the modifications can be identified through the comparison of the results before and after rehabilitation. Two cases shall be distinguished:

Case 1: The reinforcement is undertaken to reduce the bridge vibration itself. Measurement will enable the evaluation of the reduction of the vibration.

Case 2: Measurement is undertaken to evaluate the indirect effects of reinforcement by detecting the change of vibration characteristics on parts of structure.

7.3.3 Safety evaluation on a damaged bridge

In an emergency situation after an earthquake, fire, strong wind or flood, it is necessary for some bridges to be reinstated for rescue work. In this case, the remaining capacity of damaged bridges has to be evaluated quickly. After the repair of damage, measurements will be undertaken to confirm the effectiveness of the repair by evaluation of the measured results.

7.3.4 Evaluation of fatigue condition

Statistical distribution of stress for bridge elements is obtained through vibration measurement in the form of a vibration-time history. The rain flow method is used to obtain the stress range distribution and its frequency of occurrence. The evaluation of fatigue condition is undertaken mainly by using Miner’s law:

$$D = \sum_{i} n_i / N_i \tag{3}$$

where

n_i is the actual repeated cycle

N_i is the limit number to the failure for each stress range $S_i \left(i = 1, 2, ..., k\right)$, N_i is generally obtained in laboratory

In the analysis of monitored data, statistical distributions of stress, velocity, acceleration and displacement are obtained. The distributions of stress and displacement are used for condition evaluation of bridges.

7.3.5 Train running safety on rail

The safety criterion is determined by the risk of derailment of a moving train. The risk of derailment is principally evaluated using the forces acting between wheels and rails. The reduction of wheel load, magnitude of lateral thrust of the wheel and the ratio of lateral thrust and wheel load, which is called the derailment coefficient, are frequently used for evaluation. For the prevention of ballasted track instability, the bridge deck acceleration is also used.

7.4 Evaluation of serviceability

7.4.1 Serviceability evaluation on a highway bridge

The sensitivity criterion for the human body on a walk-way of a highway bridge has not been agreed internationally, but a general criterion on the human sensitivity exists (see ISO 2631-1). Depending on the problem, acceleration or velocity is measured and evaluated through comparison with the criterion.

7.4.2 Serviceability evaluation on a railway bridge

The serviceability on a railway bridge is primarily evaluated by the riding quality of passengers (see ISO 2631-4). The riding quality is usually rated using the magnitude of acceleration measured on the floor in carbodies. The vibration induced by the deflection and/or bridge end angular rotations are usually transitional, so the peak value of acceleration is suitable for evaluation.

7.4.3 Serviceability evaluation on a pedestrian bridge

Frequency and acceleration or velocity of measured response are the parameters to evaluate for the serviceability assessment of pedestrian bridges. Sensitivity criterion for human body exists as allowable level of vibration (see ISO 2631-1).

7.5 Evaluation of environmental vibration

Ground vibration and infrasound due to viaduct should be evaluated through measured data. Frequency and magnitude of the measured data should be compared with the criteria for the comfort and performance of the human body subject to vibration (see ISO 2631-1).

Annex A
(informative)

Data analysis in time and frequency domains

A.1 Expression of response in time and frequency domains

A linear system forced by an external excitation is expressed by using the condition variable as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{f} \tag{A.1}$$

where

$\boldsymbol{x}$, $\dot{\boldsymbol{x}}$ is a condition variable and its time derivative

$\boldsymbol{f}$ is the external excitation

$\boldsymbol{A}, \boldsymbol{B}$ are coefficient matrices

The equation of motion of n-th degree of freedom for excitation is given by

$$\boldsymbol{M}\ddot{\boldsymbol{y}} + \boldsymbol{C}\dot{\boldsymbol{y}} + \boldsymbol{K}\boldsymbol{y} = -\boldsymbol{M}\boldsymbol{r}\ddot{x}_g \tag{A.2}$$

where

$\boldsymbol{y}$ is the displacement vector of response

$\boldsymbol{M}$ is the mass matrix

$\boldsymbol{C}$ is the damping matrix

$\boldsymbol{K}$ is the stiffness matrix

$\boldsymbol{r}$ is the vector of influence coefficient

$\ddot{x}_g$ is the acceleration of excitation

If we replace equation (A.2) by using the relations

$$\boldsymbol{x} = \begin{bmatrix} \dot{\boldsymbol{y}} \\ \boldsymbol{y} \end{bmatrix}, \ \boldsymbol{A} = \begin{bmatrix} -\boldsymbol{M}^{-1}\boldsymbol{C} & -\mathrm{M}^{-1}\boldsymbol{K} \\ \boldsymbol{I} & \boldsymbol{0} \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} -\boldsymbol{r} \\ \boldsymbol{0} \end{bmatrix}, \ \boldsymbol{f} = \begin{bmatrix} \boldsymbol{1} \\ \boldsymbol{1} \end{bmatrix}\ddot{x}_g$$

where $\boldsymbol{I}$ is an indentity matrix and $\boldsymbol{1}$ is a column vector of ones, equation (A.2) is reduced to equation (A.1).

The impulse response of $\boldsymbol{x}$ for equation (A.1) due to an impulse input $\boldsymbol{f} = \boldsymbol{1}\delta(t - t_0)$ for equation (A.1) at $t = t_0$ is given as

$$\boldsymbol{\xi}(t - t_0) = e^{\boldsymbol{A}(t - t_0)}\boldsymbol{B}H(t - t_0) \tag{A.3}$$

where

δ is the Dirac delta function

H is the Heaviside step function

The impulse response due to $\boldsymbol{f}$ is given by

$$\boldsymbol{x}(t) = \int_{-\infty}^{\infty} \xi(t - t_0) \boldsymbol{f}(t_0) dt_0 \tag{A.4}$$

$$= \int_{-\infty}^{\infty} \xi(\tau) \boldsymbol{f}(t - \tau) d\tau$$

It is necessary to use the Duhamel integral to obtain the total response in the time domain. In order to transform the relationship between response and excitation to the frequency domain, Fourier transform on equation (A.4) is used as

$$\int_{-\infty}^{\infty} \boldsymbol{x}(t) e^{-i\omega t} dt = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \xi(\tau) \boldsymbol{f}(t - \tau) e^{-i\omega t} d\tau dt \tag{A.5}$$

$$= \int_{-\infty}^{\infty} e^{A(\tau)} \boldsymbol{B} H(\tau) e^{-i\omega\tau} d\tau \int_{-\infty}^{\infty} \boldsymbol{f}(t - \tau) e^{-i\omega\tau} d(t - \tau)$$

$$\boldsymbol{X}(\omega) = \boldsymbol{H}(\omega) \boldsymbol{F}(\omega)$$

where

$\boldsymbol{H}(\omega)$ is the Fourier transform of the impulse response function and it is called transfer function or frequency response function

By this procedure in the frequency domain, multiplication of functions can be used instead of the Duhamel integral for the time domain, as e.g. for power spectrum ***S***

$$\boldsymbol{S}_{xx}(\omega) = \boldsymbol{H}(\omega) \boldsymbol{H}(-\omega) \boldsymbol{S}_{ff}(\omega) \tag{A.6}$$

In equations (A.5) and (A.6), ***X*** and ***F*** are vectors and ***H*** is a matrix for Multiple-Input Multiple-Output (MIMO) system. If it is a Single-Input Single-Output (SISO) system, they are scalar functions

$$X(\omega) = H(\omega) F(\omega) \tag{A.7}$$

and for power and cross spectra are

$$S_{xx}(\omega) = H(\omega) H(-\omega) S_{ff}(\omega) = |H(\omega)|^2 S_{ff}(\omega) \tag{A.8}$$

$$S_{fx}(\omega) = H(\omega) S_{ff}(\omega) \tag{A.9}$$

$$S_{xx}(\omega) = H(\omega) S_{xf}(\omega) \tag{A.10}$$

In general, the vibration response $\boldsymbol{x}$ is measured and ***f*** is also measured depending on the problem. Comparison of the measured spectrum with assumed values in design and the calculation of parameter identification is done to evaluate structural condition.

A.2 Calculation of the transfer function from measured data

The transfer function $H(\omega)$ is given by using equation (A.7) as

$$H(\omega) = X(\omega)/F(\omega) \tag{A.11}$$

Attention should be payed to errors which are included in measured data. If the measured data X^* have the error $N_1(\omega)$, i.e.

$$X^*(\omega) = X(\omega) + N_1(\omega) \tag{A.12}$$

then applying equation (A.7) yields

$$H^*(\omega) = X^*(\omega)/F(\omega) = X(\omega)/F(\omega) + N_1(\omega)/F(\omega) \tag{A.13}$$

Therefore the error appears directly in the transfer function if we take simply the ratio of Fourier spectra. On the other hand, if we take the cross spectrum with F on equation (A.12), we have

$$S_{fx^*}(\omega) = S_{fx}(\omega) + S_{fn_1}(\omega) \tag{A.14}$$

When there is no correlation between input and noise, $S_{fn}(\omega) = 0$ and the transfer function $S_{fx^*}(\omega) = S_{fx}(\omega)$ is given without any effect of error as

$$H_1(\omega) = S_{fx}(\omega)/S_{ff}(\omega) \tag{A.15}$$

Similarly, if the input measurement F^* has noise effect and if it has no correlation with the output, we have the transfer function as

$$H_2(\omega) = S_{xx}(\omega)/S_{xf}(\omega) \tag{A.16}$$

And if there is no error in the measured data, the equations (A.11), (A.15) and (A.16) are the same.

$$\frac{H_1(\omega)}{H_2(\omega)} = \frac{S_{fx}(\omega)S_{xf}(\omega)}{S_{xx}(\omega)S_{ff}(\omega)} = \gamma_{xf}^2 \tag{A.17}$$

is called coherence and it takes 0 to 1 by the relationship between internal product and magnitude. If there is no noise, the estimation H_1 and H_2 take the same value and the coherence is 1. So coherence is used as a certainty index of measured data. Generally, H_1 is used as an estimation of the transfer function.

It is recommended to take the ratio of cross spectra and not to take the ratio between Fourier spectra when the transfer function is needed to be obtained from measured data.

A.3 Calculation of transfer function by analysis

The equation of motion of n-th degree of freedom is

$$\boldsymbol{M\ddot{x}} + \boldsymbol{C\dot{x}} + \boldsymbol{Kx} = \boldsymbol{f} \tag{A.18}$$

Applying the Fourier transform to both sides of the equation, we get

$$\left[-\omega^2\boldsymbol{M} - i\omega\boldsymbol{C} + \boldsymbol{K}\right]\boldsymbol{X}(\omega) = \boldsymbol{F}(\omega) \tag{A.19}$$

The transfer function between the displacement response and an external excitation is given as

$$H(\omega) = \left[-\omega^2 \boldsymbol{M} - i\omega \boldsymbol{C} + \boldsymbol{K}\right]^{-1} \tag{A.20}$$

A direct estimation of the transfer function is possible by calculating above inverse matrix. But the frequency of each mode and its contribution by direct calculation are unclear. So it is recommended to obtain the transfer function in modal space. As an example formulation of the transfer function of a non-proportionally damped system is explained. If the bridge has damping devices or isolators, the structure is considered a non-proportionally damped system. Then it is needed to use the analysis of non-proportionally damped systems as the identification of the total system of bridge.

By transformation of the variables of equation (A.18), a symmetrical ordinary differential equation is obtained:

$$\boldsymbol{A}\dot{\boldsymbol{z}} + \boldsymbol{B}\boldsymbol{z} = \boldsymbol{g} \tag{A.21}$$

where

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{M} \\ \boldsymbol{M} & \boldsymbol{0} \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} \boldsymbol{K} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K} \end{bmatrix},\ \boldsymbol{z} = \begin{Bmatrix} \boldsymbol{x} \\ \dot{\boldsymbol{x}} \end{Bmatrix},\ \boldsymbol{g} = \begin{Bmatrix} \boldsymbol{f} \\ \boldsymbol{0} \end{Bmatrix}$$

The solution of the eigenvalue calculation of the left hand side of equation (A.21) is given by complex eigenvalues and eigenvectors of 2*N*-th degree and those are the combination of conjugate values of *N*-th degree. The eigenvalues can be ordered $\lambda_j = S_j$ for $j = 1, 2, ..., N$ and $\lambda_{j+N} = S_j^*$. The eigenvectors are given as

$$\boldsymbol{V}_j = \begin{Bmatrix} \phi_j \\ S_j \phi_j \end{Bmatrix},\ \boldsymbol{V}_{j+N} = \begin{Bmatrix} \phi_j^* \\ S_j \phi_j^* \end{Bmatrix} \tag{A.22}$$

where

$$\boldsymbol{\varphi}_j = \left\{\phi_{1j}, \phi_{2j}, \cdots, \phi_{Nj}\right\}^T$$

In this calculation meaningful number of eigenvalues and modal shapes is N for S_j and ϕ_j of an N-th degree system. Frequency and modal damping are given by

$$\omega_j = \left|S_j\right| \tag{A.23}$$

$$\zeta_j = -\frac{\mathrm{Re}\left|S_j\right|}{\left|S_j\right|} \tag{A.24}$$

By normalizing the eigenvectors $\boldsymbol{V}_j$ for the matrix $\boldsymbol{A}$, the *jk* element of equation (A.20) is given as

$$H_{jk}(\omega) = \sum_{r=1}^{N} \left(\frac{\phi_{jr}\phi_{kr}}{i\omega - S_r} + \frac{\phi_{jr}^*\phi_{kr}^*}{i\omega - S_r^*} \right) \tag{A.25}$$

If the system is considered a proportionally damped system, the mode shapes are real numbers and

$$H_{jk}(\omega) = \sum_{r=1}^{N} \frac{2\phi_{jr}\phi_{kr}\left(i\omega + \zeta_r \omega_r\right)}{-\omega^2 + 2i\zeta_r\omega_r\omega + \omega_r{}^2} \tag{A.26}$$

Annex B
(informative)

Identification of vibration characteristics

B.1 Single-mode method

By assuming that the coupling effect between the different modes is small and considering the system is the sum of single-degree-of-freedom systems, frequencies and damping coefficients are obtained for each modes. Curve fitting method is useful to identify the peak of transfer function by fitting those values of 1-DOF systems. If the system has closely-spaced frequencies, identification should be done in multi-degree-of-freedom (M-DOF) system.

Half-power method to apply on the peak of transfer function and curve fitting method of modal circle on the wide range of transfer function is recommended to use.

B.2 Least square method

In order to identify the natural frequency, damping and mode shapes in frequency domain, identification of those parameters from transfer function is done to fit with the measured transfer function.

To fit the measured transfer function $H_m(\omega)$ with the assumed transfer function of non-proportionally damped system $H_c(\omega)$ for both real and imaginary part, following evaluation function

$$E = \int_{\omega_1}^{\omega_2} \left[\mathrm{Re}(H_m - H_c)^2 + \mathrm{Im}(H_m - H_c)^2 \right] W(\omega)\, \mathrm{d}\omega \tag{B.1}$$

is minimized by changing the assumed values of frequency and damping. The range ω_1 to ω_2 is frequency range for identification and $W(\omega)$ is weight coefficient to put larger values on frequency range with highly regarding in band-pass filtering.

Annex C
(informative)

Modelling of walking load

Taking the effect of both right and left steps of walking into account, walking load $f(t)$ is defined by [7] as

$$f(t) = \frac{P}{M_1}\phi(0{,}9 f_1)\sin\omega_1 t \tag{C.1}$$

where

P is the amplitude of the pulsating force

M_1 is the generalized mass for the fundamental mode

ϕ is the normalized fundamental mode shape

0,9 is the width of step, m

f_1 is the lowest natural frequency of bridge

ω_1 is the angular frequency of bridge, rad/s

t is the time

[24] defined walking load $f(t)$ as half sinusoidal function by neglecting negative load of step as

$$f(t) = F\sin\frac{\pi t}{T_C} \tag{C.2}$$

where

F is the weight of the walking body, N

T_C is the contact time of one step

t is the time

[17] defined walking load $f(t)$ by half cosine function also for positive load

$$f(t) = \alpha W\cos 2\pi f t \tag{C.3}$$

where

α is the amplification factor of load

W is the weight of the body, N

f is the walking frequency

These equations represent time variation of foot pressure on the bridge. Taking the speed and location into account, walking load is considered as moving load along the bridge.

Bibliography

[1] ISO 2631-1, *Mechanical vibration and shock — Evaluation of human exposure to whole-body vibration — Part 1: General requirements*

[2] ISO 2631-4, *Mechanical vibration and shock — Evaluation of human exposure to whole-body vibration — Part 4: Guidelines for the evaluation of the effects of vibration and rotational motion on passenger and crew comfort in fixed-guideway transport systems*

[3] ISO 8608, *Mechanical vibration — Road surface profiles — Reporting of measured data*

[4] ISO 13374-1, *Condition monitoring and diagnostics of machines — Data processing, communication and presentation — Part 1: General guidelines*

[5] ISO 13822, *Bases for design of structures — Assessment of existing structures*

[6] BS 5400-2, *Steel, concrete and composite bridges — Specification for loads*

[7] Blanchard, J., Davis, B. L., and Smith, J. W.: Design criteria and analysis for dynamic loading of foot-bridges, Proc. of Symposium of Dynamic Behaviour of Bridges, Supplementary Report 27, UK TRRL, pp. 99-106, 1977

[8] Cantieni, R.: Dynamische Belastungsversuche an Straßenbrücken in der Schweiz. 60 Jahre Erfahrung der EMPA, Eidgenössische Materialprüfungs- und Versuchsanstalt (EMPA), Massivbau, Bericht Nr. 116/1, 1983

[9] Cantieni, R.: Dynamic load testing of highway bridges, IABSE Proceedings, P-75/84, pp. 57-72, 1984

[10] Cantieni, R.: Dynamische Belastungsversuche an der Bergspurbrücke Deibüel, Teil A, Eidgenössische Materialprüfungs- und Versuchsanstalt (EMPA), Massivbau, Bericht Nr. 116/4A, 1988

[11] Chan, P. W., and Robertson, L. E.: Human perception thresholds of horizontal motion, Journal of the Structural Division ASCE, Vol. 98, No. ST8, pp. 1681-1695, 1972

[12] Cole, H. A.: On-line failure detection and damping measurement of aerospace structures by random decrement signatures, NASA CR-2205, 1973

[13] Dallard, P., et al.: The London Millennium Footbridge, The Structural Engineering, Vol. 79, No. 22, 2001

[14] Fujino, Y., et al.: Synchronization of human walking observed during lateral vibration of a congested pedestrian bridge, Earthquake Engineering and Structural Dynamics, Vol. 22, pp. 741-758, 1993

[15] Harris, C. M., and Crede, C. E.: Shock and vibration handbook, Vol. 2, McGraw-Hill Book Company, 1961

[16] Honda, H., Kajikawa, Y., and Kobori, T.: Spectra of road surface roughness on bridges, Journal of Structural Division, Proc. of ASCE, Vol. 108, No. ST9, pp. 1956-1966, 1982

[17] Kajikawa, Y.: Some considerations on ergonomical serviceability analysis of pedestrian bridge vibration, Proc. of JSCE, No. 325, pp. 23-33, 1982

[18] Kawatani, M., Nishiyama, S., and Yamada, Y.: Dynamic analysis of highway girder bridges under moving vehicles, Proc. of 4th Asia-Pacific Conference on Structural Engineering and Construction, Seoul, pp. 1857-1862, 1993

[19] Kawatani, M., and Kim, C.-H.: Effects of gap at expansion joint on traffic-induced vibration of highway bridges, Proc. of 5th International Conference on Short and Medium Span Bridges, Calgary, Canada, pp. 285-294, 1998

[20] Ljung, L.: System identification, Prentice Hall, 1987

[21] Ontario Highway Bridge Design Code 1983, Highway Engineering Division, Ministry of Transportation and Communications, OTM, 1983

[22] Pacheco, B., and Fujino, Y.: Synchronization of human walking observed during lateral vibration of a congregated bridge, Proc. of 6th US-Japan Bridge Engineering Workshop, pp. 449-462, Nevada, USA, 1990

[23] Yao, J. T. P.: Damage assessment of existing structures, Journal of Engineering Mechanics, ASCE, EM 4, 1980

[24] Wheeler, J. E.: Prediction and control of pedestrian-induced vibration in footbridges, Journal of the Structural Division, ASCE, Vol. 108, No. ST9, pp. 2045-2065, 1982